附表 1　积　分　表

说明(1)表中均省略了常数 c；(2)$\ln g(x)$均指 $\ln|g(x)|$.

一、含 $ax+b$

1. $\int \frac{1}{ax+b}\mathrm{d}x = \frac{1}{a}\ln(ax+b)$.

2. $\int \frac{1}{(ax+b)^2}\mathrm{d}x = -\frac{1}{a(ax+b)}$.

3. $\int \frac{1}{(ax+b)^3}\mathrm{d}x = -\frac{1}{2a(ax+b)^2}$.

4. $\int x(ax+b)^n\mathrm{d}x = \frac{(ax+b)^{n+2}}{a^2(n+2)} - \frac{b(ax+b)^{n+1}}{a^2(n+1)}$　$(n\neq -1,-2)$.

5. $\int \frac{x}{ax+b}\mathrm{d}x = \frac{x}{a} - \frac{b}{a^2}\ln(ax+b)$.

6. $\int \frac{x}{(ax+b)^2}\mathrm{d}x = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln(ax+b)$.

7. $\int \frac{x}{(ax+b)^3}dx = \frac{b}{2a^2(ax+b)^2} - \frac{1}{a^2(ax+b)}$.

8. $\int x^2(ax+b)^n dx = \frac{1}{a^3}\left[\frac{(ax+b)^{n+3}}{n+3} - 2b\frac{(ax+b)^{n+2}}{n+2} + b^2\frac{(ax+b)^{n+1}}{n+1}\right]$

$(n \neq -1, -2, -3)$.

9. $\int \frac{1}{x(ax+b)}dx = -\frac{1}{b}\ln\frac{ax+b}{x}$.

10. $\int \frac{1}{x^2(ax+b)}dx = -\frac{1}{bx} + \frac{a}{b^2}\ln\frac{ax+b}{x}$.

11. $\int \frac{1}{x^3(ax+b)}dx = \frac{2ax-b}{2b^2x^2} - \frac{a^2}{b^3}\ln\frac{ax+b}{x}$.

12. $\int \frac{1}{x(ax+b)^2}dx = \frac{1}{b(ax+b)} - \frac{1}{b^2}\ln\frac{ax+b}{x}$.

13. $\int \frac{1}{x(ax+b)^3}dx = \frac{1}{b^3}\left[\frac{1}{2}\left(\frac{ax+2b}{ax+b}\right)^2 - \ln\frac{ax+b}{x}\right]$.

二、含 $\sqrt{ax+b}$

14. $\int \sqrt{ax+b}dx = \frac{2}{3a}\sqrt{(ax+b)^3}$.

15. $\int x\sqrt{ax+b}dx = \frac{2(3ax-2b)}{15a^2}\sqrt{(ax+b)^3}$.

16. $\int x^2\sqrt{ax+b}dx = \frac{2(15a^2x^2-12abx+8b^2)}{105a^3}\sqrt{(ax+b)^3}$.

17. $\int x^n\sqrt{ax+b}dx = \frac{2x^n}{(2n+3)a}\sqrt{(ax+b)^3} - \frac{2nb}{(2n+3)a}\int x^{n-1}\sqrt{ax+b}dx$.

18. $\int \frac{1}{\sqrt{ax+b}}dx = \frac{2}{a}\sqrt{ax+b}$.

19. $\int \frac{x}{\sqrt{ax+b}}dx = \frac{2(ax-2b)}{3a^2}\sqrt{ax+b}$.

20. $\int \frac{x^n}{\sqrt{ax+b}}dx = \frac{2x^n}{(2n+1)a}\sqrt{ax+b} - \frac{2nb}{(2n+1)a}\int \frac{x^{n-1}}{\sqrt{ax+b}}dx$.

21. $\int \frac{1}{x\sqrt{ax+b}}dx = \frac{1}{\sqrt{b}}\ln\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}$ $(b>0)$.

22. $\int \frac{1}{x\sqrt{ax+b}}dx = \frac{2}{\sqrt{-b}}\arctan\sqrt{\frac{ax+b}{-b}}$ $(b<0)$.

23. $\int \frac{1}{x^n\sqrt{ax+b}}dx = -\frac{\sqrt{ax+b}}{(n-1)bx^{n-1}} - \frac{(2n-3)a}{2(n-1)b}\int \frac{dx}{x^{n-1}\sqrt{ax+b}} \quad (n>1).$

24. $\int \frac{\sqrt{ax+b}}{x}dx = 2\sqrt{ax+b} + b\int \frac{1}{x\sqrt{ax+b}}dx.$

25. $\int \frac{\sqrt{ax+b}}{x^n}dx = -\frac{\sqrt{(ax+b)^3}}{(n-1)bx^{n-1}} - \frac{(2n-5)a}{2(n-1)b}\int \frac{\sqrt{ax+b}}{x^{n-1}}dx \quad (n>1).$

26. $\int x\sqrt{(ax+b)^n}dx = \frac{2}{a^2}\left[\frac{1}{n+4}\sqrt{(ax+b)^{n+4}} - \frac{b}{n+2}\sqrt{(ax+b)^{n+2}}\right].$

27. $\int \frac{x}{\sqrt{(ax+b)^n}}dx = \frac{2}{a^2}\left[\frac{b}{n-2}\frac{1}{\sqrt{(ax+b)^{n-2}}} - \frac{1}{n-4}\frac{1}{\sqrt{(ax+b)^{n-4}}}\right].$

三、含 $\sqrt{ax+b}$, $\sqrt{cx+d}$

28. $\int \frac{1}{\sqrt{ax+b}\sqrt{cx+d}}dx = \frac{2}{\sqrt{ac}}\operatorname{artanh}\sqrt{\frac{c(ax+b)}{a(cx+d)}} \quad (ac>0).$

29. $\int \frac{1}{\sqrt{ax+b}\sqrt{cx+d}}dx = \frac{2}{\sqrt{-ac}}\arctan\sqrt{\frac{-c(ax+b)}{a(cx+d)}} \quad (ac<0).$

30. $\int \sqrt{ax+b}\sqrt{cx+d}dx = \frac{2acx+ad+bc}{4ac}\sqrt{ax+b}\sqrt{cx+d} - \frac{(ad-bc)^2}{8ac}\int \frac{dx}{\sqrt{ax+b}\cdot\sqrt{cx+d}}.$

31. $\int \sqrt{\frac{ax+b}{cx+d}}dx = \frac{\sqrt{ax+b}\sqrt{cx+d}}{c} - \frac{ad-bc}{2c}\int \frac{dx}{\sqrt{ax+b}\sqrt{cx+d}}.$

32. $\int \frac{1}{\sqrt{(x-p)(q-x)}}dx = 2\arcsin\sqrt{\frac{x-p}{q-p}}.$

四、含 ax^2+c

33. $\int \frac{1}{ax^2+c}dx = \frac{1}{\sqrt{ac}}\arctan\left(x\sqrt{\frac{a}{c}}\right) \quad (a>0, c>0).$

34. $\int \frac{1}{ax^2+c}dx = \frac{1}{2\sqrt{-ac}}\ln\frac{x\sqrt{a}-\sqrt{-c}}{x\sqrt{a}+\sqrt{-c}} \quad (a>0, c<0).$

$\int \frac{1}{ax^2+c}dx = \frac{1}{2\sqrt{-ac}}\ln\frac{\sqrt{c}+x\sqrt{-a}}{\sqrt{c}-x\sqrt{-a}} \quad (a<0, c>0).$

35. $\int \frac{1}{(ax^2+c)^n}dx = \frac{x}{2c(n-1)(ax^2+c)^{n-1}} + \frac{2n-3}{2c(n-1)}\int \frac{dx}{(ax^2+c)^{n-1}} \quad (n>1).$

36. $\int x(ax^2+c)^n dx = \frac{(ax^2+c)^{n+1}}{2a(n+1)} \quad (n\neq -1).$

37. $\int \frac{x}{ax^2+c}dx = \frac{1}{2a}\ln(ax^2+c).$

38. $\int \frac{x^2}{ax^2+c}dx = \frac{x}{a} - \frac{c}{a}\int \frac{dx}{ax^2+c}.$

39. $\int \frac{x^n}{ax^2+c}dx = \frac{x^{n-1}}{a(n-1)} - \frac{c}{a}\int \frac{x^{n-2}}{ax^2+c}dx \quad (n\neq -1).$

五、含 $\sqrt{ax^2+c}$

40. $\int \sqrt{ax^2+c}dx = \frac{x}{2}\sqrt{ax^2+c} + \frac{c}{2\sqrt{a}}\ln(x\sqrt{a} + \sqrt{ax^2+c}) \quad (a>0).$

41. $\int \sqrt{ax^2+c}dx = \frac{x}{2}\sqrt{ax^2+c} + \frac{c}{2\sqrt{-a}}\arcsin\left(x\sqrt{\frac{-a}{c}}\right) \quad (a<0).$

42. $\int \sqrt{(ax^2+c)^3}dx = \frac{x}{8}(2ax^2+5c)\sqrt{ax^2+c} + \frac{3c^2}{8\sqrt{a}}\ln(x\sqrt{a} + \sqrt{ax^2+c}). \quad (a>0).$

43. $\int \sqrt{(ax^2+c)^3}dx = \frac{x}{8}(2a^2x+5c)\sqrt{ax^2+c} + \frac{3c^2}{8\sqrt{-a}}\arcsin\left(x\sqrt{\frac{-a}{c}}\right)$ $(a<0).$

44. $\int x\sqrt{ax^2+c}dx = \frac{1}{3a}\sqrt{(ax^2+c)^3}.$

45. $\int x^2\sqrt{ax^2+c}dx = \frac{x}{4a}\sqrt{(ax^2+c)^3} - \frac{cx}{8a}\sqrt{ax^2+c} - \frac{c^2}{8\sqrt{a^3}}\ln(x\sqrt{a} + \sqrt{ax^2+c}) \quad (a>0).$

46. $\int x^2\sqrt{ax^2+c}dx = \frac{x}{4a}\sqrt{(ax^2+c)^3} - \frac{cx}{8a}\sqrt{ax^2+c} - \frac{c^2}{8a\sqrt{-a}}\arcsin\left(x\sqrt{\frac{-a}{c}}\right) \quad (a<0).$

47. $\int x^n\sqrt{ax^2+c}dx = \frac{x^{n-1}}{(n+2)a}\sqrt{(ax^2+c)^3} - \frac{(x-1)c}{(n+2)a}\int x^{n-2}\sqrt{ax^2+c}dx$ $(n>0).$

48. $\int x\sqrt{(ax^2+c)^3}\mathrm{d}x=\frac{1}{5a}\sqrt{(ax^2+c)^5}$.

49. $\int x^2\sqrt{(ax^2+c)^3}\mathrm{d}x=\frac{x^3}{6}\sqrt{(ax^2+c)^3}+\frac{c}{2}\int x^2\sqrt{ax^2+c}\mathrm{d}x$.

50. $\int x^n\sqrt{(ax^2+c)^3}\mathrm{d}x=\frac{x^{n+1}}{n+4}\sqrt{(ax^2+c)^3}+\frac{3c}{n+4}\int x^n\sqrt{ax^2+c}\mathrm{d}x$.
$(n>0)$.

51. $\int\frac{\sqrt{ax^2+c}}{x}\mathrm{d}x=\sqrt{ax^2+c}+\sqrt{c}\ln\frac{\sqrt{ax^2+c}-\sqrt{c}}{x}$ $(c>0)$.

52. $\int\frac{\sqrt{ax^2+c}}{x}\mathrm{d}x=\sqrt{ax^2+c}-\sqrt{-c}\arctan\frac{\sqrt{ax^2+c}}{\sqrt{-c}}$, $(c<0)$.

53. $\int\frac{\sqrt{ax^2+c}}{x^n}\mathrm{d}x=-\frac{\sqrt{(ax^2+c)^3}}{c(n-1)x^{n-1}}-\frac{(n-4)a}{(n-1)c}\int\frac{\sqrt{ax^2+c}}{x^{n-2}}\mathrm{d}x$ $(n>1)$.

54. $\int\frac{\mathrm{d}x}{\sqrt{ax^2+c}}=\frac{1}{\sqrt{a}}\ln(x\sqrt{a}+\sqrt{ax^2+c})$ $(a>0)$.

55. $\int\frac{\mathrm{d}x}{\sqrt{ax^2+c}}=\frac{1}{\sqrt{-a}}\arcsin\left(x\sqrt{\frac{-a}{c}}\right)$ $(a<0)$.

56. $\int\frac{\mathrm{d}x}{\sqrt{(ax^2+c)^3}}=\frac{x}{c\sqrt{ax^2+c}}$.

57. $\int\frac{x}{\sqrt{ax^2+c}}\mathrm{d}x=\frac{1}{a}\sqrt{ax^2+c}$.

58. $\int\frac{x^2}{\sqrt{ax^2+c}}\mathrm{d}x=\frac{x}{a}\sqrt{ax^2+c}-\frac{1}{a}\int\sqrt{ax^2+c}\mathrm{d}x$.

59. $\int\frac{x^n}{\sqrt{ax^2+c}}\mathrm{d}x=\frac{x^{n-1}}{na}\sqrt{ax^2+c}-\frac{(n-1)c}{na}\int\frac{x^{n-2}}{\sqrt{ax^2+c}}\mathrm{d}x$ $(n>0)$.

60. $\int\frac{1}{x\sqrt{ax^2+c}}\mathrm{d}x=\frac{1}{\sqrt{c}}\ln\frac{\sqrt{ax^2+c}-\sqrt{c}}{x}$ $(c>0)$.

61. $\int\frac{1}{x\sqrt{ax^2+c}}\mathrm{d}x=\frac{1}{\sqrt{-c}}\operatorname{arcsec}\left(x\sqrt{\frac{-a}{c}}\right)$ $(c<0)$.

62. $\int\frac{1}{x^2\sqrt{ax^2+c}}\mathrm{d}x=-\frac{\sqrt{ax^2+c}}{cx}$.

63. $\int\frac{1}{x^n\sqrt{ax^2+c}}\mathrm{d}x=-\frac{\sqrt{ax^2+c}}{c(n-1)x^{n-1}}-\frac{(n-2)a}{(n-1)c}\int\frac{\mathrm{d}x}{x^{n-2}\sqrt{ax^2+c}}$ $(n>1)$.

六、含 ax^2+bx+c

64. $\int \frac{1}{ax^2+bx+c}dx = \frac{1}{\sqrt{b^2-4ac}} \ln \frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}$　$(b^2>4ac)$.

65. $\int \frac{1}{ax^2+bx+c}dx = \frac{2}{\sqrt{4ac-b^2}} \arctan \frac{2ax+b}{\sqrt{4ac-b^2}}$　$(b^2<4ac)$.

66. $\int \frac{1}{ax^2+bx+c}dx = -\frac{2}{2ax+b}$.　$(b^2=4ac)$.

67. $\int \frac{1}{(ax^2+bx+c)^n}dx = \frac{2ax+b}{(n-1)(4ac-b^2)(ax^2+bx+c)^{n-1}} + \frac{2(2n-3)a}{(n-1)(4ac-b^2)} \int \frac{dx}{(ax^2+bx+c)^{n-1}}$　$(n>1, b^2 \neq 4ac)$.

68. $\int \frac{x}{ax^2+bx+c}dx = \frac{1}{2a} \ln(ax^2+bx+c) - \frac{b}{2a} \int \frac{dx}{ax^2+bx+c}$.

69. $\int \frac{x^2}{ax^2+bx+c}dx = \frac{x}{a} - \frac{b}{2a^2} \ln(ax^2+bx+c) + \frac{b^2-2ac}{2a^2} \int \frac{dx}{ax^2+bx+c}$.

70. $\int \frac{x^n}{ax^2+bx+c}dx = \frac{x^{n-1}}{(n-1)a} - \frac{c}{a} \int \frac{x^{n-2}}{ax^2+bx+c}dx - \frac{b}{a} \int \frac{x^{n-1}}{ax^2+bx+c}dx$　$(n>1)$.

七、含 $\sqrt{ax^2+bx+c}$

71. $\int \frac{1}{\sqrt{ax^2+bx+c}}dx = \frac{1}{\sqrt{a}} \ln(2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c})$　$(a>0)$.

72. $\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{-a}} \arcsin \frac{-2ax-b}{\sqrt{b^2-4ac}}$　$(a<0, b^2>4ac)$.

73. $\int \frac{x dx}{\sqrt{ax^2+bx+c}} = \frac{\sqrt{ax^2+bx+c}}{a} - \frac{b}{2a} \int \frac{dx}{\sqrt{ax^2+bx+c}}$.

74. $\int \frac{x^n dx}{\sqrt{ax^2+bx+c}} = \frac{x^{n-1}}{na}\sqrt{ax^2+bx+c} - \frac{(2n-1)b}{2na} \int \frac{x^{n-1}}{\sqrt{ax^2+bx+c}}dx - \frac{(n+1)c}{na} \int \frac{x^{n-2}}{\sqrt{ax^2+bx+c}}dx$.

75. $\int \sqrt{ax^2+bx+c}dx = \frac{2ax+b}{4a}\sqrt{ax^2+bx+c} - \frac{b^2-4ac}{8a} \int \frac{dx}{\sqrt{ax^2+bx+c}}$.

76. $\int x\sqrt{ax^2+bx+c}dx = \frac{1}{3a}\sqrt{(ax^2+bx+c)^3} - \frac{b}{2a} \int \sqrt{ax^2+bx+c}\,dx$.

77. $\int x^2\sqrt{ax^2+bx+c}dx = \left(x-\frac{5b}{6a}\right)\frac{\sqrt{(ax^2+bx+c)^3}}{4a} +$

$$\frac{5b^2-4ac}{16a^2}\int\sqrt{ax^2+bx+c}\,dx.$$

78. $$\int\frac{1}{x\sqrt{ax^2+bx+c}}dx=-\frac{1}{\sqrt{c}}\ln\left(\frac{\sqrt{ax^2+bx+c}+\sqrt{c}}{x}+\frac{b}{2\sqrt{c}}\right)\quad(c>0).$$

79. $$\int\frac{1}{x\sqrt{ax^2+bx+c}}dx=\frac{1}{\sqrt{-c}}\arcsin\frac{bx+2c}{x\sqrt{b^2-4ac}}\quad(c<0,b^2>4ac).$$

80. $$\int\frac{dx}{x\sqrt{ax^2+bx}}=-\frac{2}{bx}\sqrt{ax^2+bx}.$$

81. $$\int\frac{dx}{x^n\sqrt{ax^2+bx+c}}=-\frac{\sqrt{ax^2+bx+c}}{(n-1)cx^{n-1}}-\frac{(2n-3)b}{2(n-1)c}\int\frac{dx}{x^{n-1}\sqrt{ax^2+bx+c}}-\frac{(n-2)a}{(n-1)c}\int\frac{dx}{x^{n-2}\sqrt{ax^2+bx+c}}\quad(n>1).$$

八、含 $\sin ax$

82. $$\int\sin ax\,dx=-\frac{1}{a}\cos ax.$$

83. $$\int\sin^2 ax\,dx=\frac{x}{2}-\frac{1}{4a}\sin 2ax.$$

84. $$\int\sin^3 ax\,dx=-\frac{1}{a}\cos ax+\frac{1}{3a}\cos^3 ax.$$

85. $$\int\sin^n ax\,dx=-\frac{1}{na}\sin^{n-1}ax\cos ax+\frac{n-1}{n}\int\sin^{n-2}ax\,dx\quad(n\text{ 为正整数}).$$

86. $$\int\frac{1}{\sin ax}dx=\frac{1}{a}\ln\tan\frac{ax}{2}.$$

87. $$\int\frac{1}{\sin^2 ax}dx=-\frac{1}{a}\cot ax.$$

88. $$\int\frac{1}{\sin^n ax}dx=-\frac{\cos ax}{(n-1)a\sin^{n-1}ax}+\frac{n-2}{n-1}\int\frac{dx}{\sin^{n-2}ax}.\quad(n\geqslant 2\text{ 为整数}).$$

89. $$\int\frac{dx}{1\pm\sin ax}=\mp\frac{1}{a}\tan\left(\frac{\pi}{4}\mp\frac{ax}{2}\right).$$

90. $$\int\frac{dx}{b+c\sin ax}=-\frac{2}{a\sqrt{b^2-c^2}}\arctan\left[\sqrt{\frac{b-c}{b+c}}\tan\left(\frac{\pi}{4}-\frac{ax}{2}\right)\right]\quad(b^2>c^2).$$

91. $$\int\frac{dx}{b+c\sin ax}=-\frac{1}{a\sqrt{c^2-b^2}}\ln\frac{c+b\sin ax+\sqrt{c^2-b^2}\cos ax}{b+c\sin ax}\quad(b^2<c^2).$$

92. $$\int\sin ax\sin bx\,dx=\frac{\sin(a-b)x}{2(a-b)}-\frac{\sin(a+b)x}{2(a+b)},\ |a|\neq|b|.$$

九、含 $\cos ax$

93. $\int \cos ax\,dx = \frac{1}{a}\sin ax.$

94. $\int \cos^2 ax\,dx = \frac{x}{2} + \frac{1}{4a}\sin 2ax.$

95. $\int \cos^n ax\,dx = \frac{1}{na}\cos^{n-1} ax \sin ax + \frac{n-1}{n}\int \cos^{n-2} ax\,dx$　（n 为正整数）.

96. $\int \frac{1}{\cos ax}dx = \frac{1}{a}\ln\tan\left(\frac{\pi}{4} + \frac{ax}{2}\right).$

97. $\int \frac{1}{\cos^2 ax}dx = \frac{1}{a}\tan ax.$

98. $\int \frac{1}{\cos^n ax}dx = \frac{\sin ax}{(n-1)a\cos^{n-1} ax} + \frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2} ax}$　（$n \geqslant 2$ 为整数）.

99. $\int \frac{dx}{1+\cos ax} = \frac{1}{a}\tan\frac{ax}{2}.$

100. $\int \frac{dx}{1-\cos ax} = -\frac{1}{a}\cot\frac{ax}{2}.$

101. $\int \frac{dx}{b+c\cos ax} = \frac{1}{a\sqrt{b^2-c^2}}\arctan\frac{\sqrt{b^2-c^2}\sin ax}{c+b\cos ax}$　（$|b|>|c|$）.

102. $\int \frac{dx}{b+c\cos ax} = \frac{1}{c-b}\sqrt{\frac{c-b}{c+b}}\ln\frac{\tan\frac{x}{2} + \sqrt{\frac{c+b}{c-b}}}{\tan\frac{x}{2} - \sqrt{\frac{c+b}{c-b}}}$　（$|b|<|c|$）.

103. $\int \cos ax\cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)}$　（$|a|\neq|b|$）.

十、含 $\sin ax$ 和 $\cos ax$

104. $\int \sin ax\cos bx\,dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}$　（$|a|\neq|b|$）.

105. $\int \sin^n ax\cos ax\,dx = \frac{1}{(n+1)a}\sin^{n+1} ax$　（$n\neq -1$）.

106. $\int \sin ax\cos^n ax\,dx = -\frac{1}{(n+1)a}\cos^{n+1} ax$　（$n\neq -1$）.

107. $\int \frac{\sin ax}{\cos ax}dx = -\frac{1}{a}\ln\cos ax.$

108. $\int \frac{\cos ax}{\sin ax}dx = \frac{1}{a}\ln\sin ax.$

109. $\int \frac{dx}{b^2\cos^2 ax + c^2\sin^2 ax} = \frac{1}{abc}\arctan\frac{c\cdot\tan ax}{b}.$

110. $\int \sin^2 ax\cos^2 ax\,dx = \frac{x}{8} - \frac{1}{32a}\sin 4ax.$

111. $\int \frac{dx}{\sin ax\cos ax} = \frac{1}{a}\ln\tan ax.$

112. $\int \frac{dx}{\sin^2 ax\cos^2 ax} = \frac{1}{a}(\tan ax - \cot ax).$

113. $\int \frac{\sin^2 ax}{\cos ax}dx = -\frac{1}{a}\sin ax + \frac{1}{a}\ln\tan\left(\frac{\pi}{4} + \frac{ax}{2}\right).$

114. $\int \frac{\cos^2 ax}{\sin ax}dx = \frac{1}{a}\cos ax + \frac{1}{a}\ln\tan\frac{ax}{2}.$

115. $\int \frac{\cos ax}{b + c\sin ax}dx = \frac{1}{ac}\ln(b + c\sin ax).$

116. $\int \frac{\sin ax}{b + c\cos ax}dx = -\frac{1}{ac}\ln(b + c\cos ax).$

117. $\int \frac{dx}{b\sin ax + c\cos ax} = \frac{1}{a\sqrt{b^2 + c^2}}\ln\tan\frac{ax + \arctan\frac{c}{b}}{2}.$

十一、含 $\tan ax, \cot ax$

118. $\int \tan ax\,dx = -\frac{1}{a}\ln\cos ax.$

119. $\int \cot ax\,dx = \frac{1}{a}\ln\sin ax.$

120. $\int \tan^2 ax\,dx = \frac{1}{a}\tan ax - x.$

121. $\int \cot^2 ax\,dx = -\frac{1}{a}\cot ax - x.$

122. $\int \tan^n ax\,dx = \frac{1}{(n-1)a}\tan^{n-1} ax - \int \tan^{n-2} ax\,dx$　$(n \geqslant 2$ 为整数$)$.

123. $\int \cot^n ax\,dx = -\frac{1}{(n-1)a}\cot^{n-1} ax - \int \cot^{n-2} ax\,dx$　$(n \geqslant 2$ 为整数$)$.

十二、含 $x^n \sin ax, x^n \cos ax$

124. $\int x\sin ax\,dx = \frac{1}{a^2}\sin ax - \frac{1}{a}x\cos ax.$

125. $\int x^2\sin ax\,dx = \frac{2x}{a^2}\sin ax + \frac{2}{a^3}\cos ax - \frac{x^2}{a}\cos ax.$

126. $\int x^n \sin ax\,dx = -\frac{x^n}{a}\cos ax + \frac{n}{a}\int x^{n-1}\cos ax\,dx$.

127. $\int x\cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$.

128. $\int x^2\cos ax\,dx = \frac{2x}{a^2}\cos ax - \frac{2}{a^3}\sin ax + \frac{x^2}{a}\sin ax$.

129. $\int x^n\cos ax\,dx = \frac{x^n}{a}\sin ax - \frac{n}{a}\int x^{n-1}\sin ax\,dx \quad (n>0)$.

十三、含 e^{ax}

130. $\int e^{ax}\,dx = \frac{1}{a}e^{ax}$.

131. $\int b^{ax}\,dx = \frac{1}{a\ln b}b^{ax}$.

132. $\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1)$.

133. $\int xb^{ax}\,dx = \frac{xb^{ax}}{a\ln b} - \frac{b^{ax}}{a^2(\ln b)^2}$.

134. $\int x^n e^{ax} = \frac{e^{ax}}{a^{n+1}}[(ax)^n - n(ax)^{n-1} + n(n-1)(ax)^{n-2} + \cdots + (-1)^n n!]$

(n 为正整数).

135. $\int x^n b^{ax}\,dx = \frac{x^n b^{ax}}{a\ln b} - \frac{n}{a\ln b}\int x^{n-1}b^{ax}\,dx \quad (n>0)$.

136. $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$.

137. $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$.

十四、含 $\ln ax$

138. $\int \ln ax\,dx = x\ln ax - x$.

139. $\int x\ln ax\,dx = \frac{x^2}{2}\ln ax - \frac{x^2}{4}$.

140. $\int x^n \ln ax\,dx = \frac{x^{n+1}}{n+1}\ln ax - \frac{x^{n+1}}{(n+1)^2} \quad (n\neq -1)$.

141. $\int \frac{1}{x\ln ax}dx = \ln\ln ax$.

142. $\int \frac{1}{x(\ln ax)^n}dx = -\frac{1}{(n-1)(\ln ax)^{n-1}} \quad (n\neq 1)$.

143. $\int \frac{x^n}{(\ln ax)^m}\mathrm{d}x = -\frac{x^{n+1}}{(m-1)(\ln ax)^{m-1}} + \frac{n+1}{m-1}\int \frac{x^n}{(\ln ax)^{m-1}}\mathrm{d}x \quad (m \neq 1)$.

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144. $\int \arcsin ax\,\mathrm{d}x = x\arcsin ax + \frac{1}{a}\sqrt{1-a^2x^2}$.

145. $\int (\arcsin ax)^2\mathrm{d}x = x(\arcsin ax)^2 - 2x + \frac{2}{a}\sqrt{1-a^2x^2}\arcsin ax$.

146. $\int x\arcsin ax\,\mathrm{d}x = \left(\frac{x^2}{2} - \frac{1}{4a^2}\right)\arcsin ax + \frac{x}{4a}\sqrt{1-a^2x^2}$.

147. $\int \arccos ax\,\mathrm{d}x = x\arccos ax - \frac{1}{a}\sqrt{1-a^2x^2}$.

148. $\int (\arccos ax)^2\mathrm{d}x = x(\arccos ax)^2 - 2x - \frac{2}{a}\sqrt{1-a^2x^2}\arccos ax$.

149. $\int x\arccos ax\,\mathrm{d}x = \left(\frac{x^2}{2} - \frac{1}{4a^2}\right)\arccos ax - \frac{x}{4a}\sqrt{1-a^2x^2}$.

150. $\int \arctan ax\,\mathrm{d}x = x\arctan ax - \frac{1}{2a}\ln(1+a^2x^2)$.

151. $\int x^n \arctan ax\,\mathrm{d}x = \frac{x^{n+1}}{n+1}\arctan ax - \frac{a}{n+1}\int \frac{x^{n+1}}{1+a^2x^2}\mathrm{d}x \quad (n \neq -1)$.

152. $\int \operatorname{arccot} ax\,\mathrm{d}x = x\operatorname{arccot} ax + \frac{1}{2a}\ln(1+a^2x^2)$.

153. $\int x^n \operatorname{arccot} ax\,\mathrm{d}x = \frac{x^{n+1}}{n+1}\operatorname{arccot} ax + \frac{a}{n+1}\int \frac{x^{n+1}}{1+a^2x^2}\mathrm{d}x \quad (n \neq -1)$.